THE GALACTIC DISK INFLUENCE ON THE OORT CLOUD
COMETARY ORBITS

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Abstract. The long term evolution of the Oort cloud cometary orbits under the influence of the Galactic tidal force was studied by exact numerical integration of motion. Two different types of evolution are found dividing the Oort cloud orbits into two separate classes. The necessary conditions to obtain observable orbits are also discussed.

1. Introduction

The influence of galactic perturbations on the cometary motion in the Oort cloud was first introduced only thirty years ago. Several new papers concerning this subject have appeared lately, for example: Heisler and Tremaine (1986), Heisler (1990), Byl (1983, 1986,1990), Matese and Whitman (1989, 1992). Among the results they repeat the conclusion that the influence of the Galactic Disk is about ten times stronger than the influence of the Galactic Center. The tidal effect of the galactic disk is widely considered as a main source of perturbations on cometary orbits in the Oort cloud. In all papers published until now the tidal effect was studied on rather short intervals of time (from 1 to tens of orbital revolutions), and the results were often obtained after significant simplifications of analytical nature adopted for the tractability of the problem. The subject discussed here remains in close connection with several classical questions of cometary astronomy, such as the asymmetry of the orbital perihelion distribution or the production of observable comets from the Oort cloud. We present some results of long term orbital evolution modelling (up to thousands of revolutions) under the influence of galactic disk perturbations.
2. Dynamic Model

We numerically integrate the equations of motion proposed by Harrington (1985) and widely used up to the present:

\[
\begin{align*}
\ddot{x} &= -\frac{\mu}{r^3}x \\
\ddot{y} &= -\frac{\mu}{r^3}y \\
\ddot{z} &= -\frac{\mu}{r^3}z - 4\pi G \rho \cdot z
\end{align*}
\]

where \(4\pi G \rho \cdot z\) describes the disturbing force perpendicular to the galactic disk and \(\rho\) is the mean density in the Solar neighbourhood, equal to \(0.185M_\odot/pc^3\) (Bahcall 1984).

![Figure 1](image)

Fig. 1. Orbital elements evolution for class A orbit: time variation of the perihelion distance \(q\), and the eccentricity \(e\).

We describe the cometary orbit by classical parameters: perihelion distance \(q\), aphelion distance \(Q\), argument of perihelion \(\omega\), longitude of the ascending node \(\Omega\), inclination of the orbit \(i\) and additionally the inclination of the line of apses \(b\). All angular elements are measured with respect to the galactic disk plane. Element \(b\) can be obtained from the simple relation:

\[
\sin(b) = \sin(i) \cdot \sin(\omega).
\]
The presented equations were integrated by two alternative methods, namely the Gauss-Radau method (the well known RA15 routine by Everhart 1985) and the seventh order Runge-Kutta Nystrom method (D2RKD7 routine from Fox 1984) on time intervals of the order of 5 billion years. The results are presented as plots of time evolution of all orbital elements. The detailed description of both, the model and the calculation methods, as well as a broad discussion of the results may be found in Prętka (1993).

3. Results

We analyzed the evolution of several thousand cometary orbits with parameters from the following intervals: \( q \in [500\text{AU},50000\text{AU}] \), \( Q \in [1000\text{AU},80000\text{AU}] \) and angular elements uniformly distributed over the celestial sphere. Basing on this investigation one can formulate several general conclusions:

1. Orbits with zero inclination are not perturbed, which follows directly from the equations of motion.

2. All changes of the elements are strictly periodic, superposed are long-term waves of large amplitudes and short-period ones with amplitudes tens of times smaller (well observed in Fig.3 and Fig.4).

![Fig. 2. Orbital elements evolution for class A orbit: time variation of the inclination of the orbit \( i \), and the line of apses inclination \( b \).](image)

3. The population of cometary orbits may be divided into two classes: A and
B with respect to the shape of the evolutionary curves. The division is defined by the critical value of the line of apses inclination $b^*$ equal to $25^\circ - 29^\circ$, depending on the semimajor axis and the eccentricity of the given orbit.

Class A. When $b < b^*$ the line of apses never goes farther from the galactic plane and oscillates about it. The inclination of such an orbit decreases to the minimal value when the absolute value of $b$ is the largest. In the same moment one can notice the fastest changes of the longitude of the ascending node and the argument of perihelion. These two elements show cyclic changes from $0^\circ - 360^\circ$ (an example of this kind of orbital evolution is shown in Fig.1 and Fig.2).

Class B. For orbits from the second group $b$ is always greater than $b^*$, reaching the largest value which may be close to $90^\circ$. Now, the inclination attains its lowest value when $b$ is the smallest. Again, the fastest changes of the rest of the angular elements are observed in the same moment. The argument of perihelion $\omega$ now oscillates in a symmetric interval centered on $90^\circ$ (an example of this kind of orbital evolution is shown in Fig.3 and Fig.4).

Fig. 3. Orbital elements evolution for class B orbit: time variation of the perihelion distance $q$, and the eccentricity $e$.

4. Production of Observable Orbits

Observable orbits may occur when the amplitude of variation in the perihelion distance is large enough to decrease it below the 5 AU limit. This means that in the
absence of planetary perturbations such a comet could enter the Solar neighbour-
hood several times during the lifetime of the Solar System (up to several hundred
times depending on the size and the shape of the orbit). An example of such a situa-
tion is presented in Fig.3 and Fig.4. The perihelion distance decreases in the fastest
manner when the initial value of the inclination is close to 90° and only in this case
does it go below the observability limit. When exactly equal to 90°, the inclination
remains constant. Otherwise it decreases rapidly when the perihelion distance rea-
ches its minimum. From the coincidence of minima of the perihelion distance and
the inclination with the line of apses inclination close to the critical value $b^*$ for
the given orbit an interesting conclusion appears: galactic disk tidal perturbations
cannot produce an observable cometary orbit with an inclination lower than the
critical value of the parameter $b$. Since the semimajor axis is perturbed very slightly
the eccentricity of all observable orbits is very close to unity.

![Graph](image)

**Fig. 4.** Orbital elements evolution for class B orbit: time variation of the inclination of
the orbit $i$, and the line of apses inclination $b$.

From the strictly periodic character of all element changes it follows that it is im-
possible to observe parabolic or hyperbolic orbits produced from an initially elliptic
one in the Oort cloud by galactic disk perturbations. Therefore these perturbati-
ons cannot increase or decrease the overall number of cometary orbits bound to
the Sun.
5. Future Plans

As it has already been emphasized, the presented evolution of orbital elements has been obtained by exact numerical integration of the equations of motion shown above. In the latest papers (for example: Matese and Whitman 1989, 1992) one can find analytical solutions of the averaged equations. Numerical integration is a very time consuming method and we plan to perform the comparison of the accuracy and the efficiency of three different methods of solving the problem before we start statistical investigations. These three methods are: direct numerical integration in rectangular coordinates, numerical integration of averaged equations or use of approximate analytical solutions (Breiter 1993). The optimal choice of the method allows us to obtain a statistical picture of galactic disk tidal perturbational effects by the Monte Carlo simulation of the Oort cloud dynamics. It might be interesting to investigate both cometary population characteristics in the cloud and statistical properties of the population of observable comets produced by the galactic disk perturbations. As the next step one may extend the dynamic model taking into account the influence of the galactic center and stellar perturbations.

References

Harrington, R.S.: 1985, “Implications of the observed distributions of very long period comet orbits”. Icarus, 61, 60-62.