THE STATISTICAL EFFECTS OF GALACTIC TIDES ON THE OORT CLOUD

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Abstract. This report is a comment on two papers by Matese and Whitman (1989, 1992). We discuss here the applicability of uniform probability densities for the orbital parameters of the Oort cloud comets.

Key words: Galactic tides – Oort Cloud – comets

1. Introduction

In this work we concentrate on the classical picture of the galactic tides where the local matter density in the galaxy is constant in time. A review of selected papers on galactic tides and a detailed description of the adopted dynamical model can be found in Prętka and Dybczyński (1994, Paper A). The equations of motion in that model are:

$$\ddot{x} = -\frac{\mu}{r^3}x \ , \qquad \ddot{y} = -\frac{\mu}{r^3}y \ , \qquad \ddot{z} = -\frac{\mu}{r^3}z - 4\pi G\rho \cdot z \tag{1}$$

with $\rho = 0.185 M_\odot/pc^3$ (Bahcall 1984), see also Heisler (1990). We also presented there examples of a long-term orbital evolution obtained from numerical integration of those equations, demonstrating the superposition of a long term, strictly periodic variation (with period of order of several hundreds of orbital revolutions) and local short period fluctuations in orbital elements. These short period terms are caused by perturbations in semimajor axis and have a period comparable with orbital one.

The galactic tidal effect can be also studied using secularly averaged Hamiltonian equations. These can be expressed in terms of the Delaunay coordinates and momenta as was done, for example, in two papers by Matese and Whitman (1989, 1992), who provided an analytical solution of the problem.

To test the applicability of the first order secularly averaged Hamiltonian technique to the galactic tide effect, we compare the results obtained with that method to the numerical integration described above. The results are in good agreement even over long time intervals, except for the absence of the short period fluctuations what is obvious result of averaging technique (semimajor axis is constant in this case). The long term periodic variation

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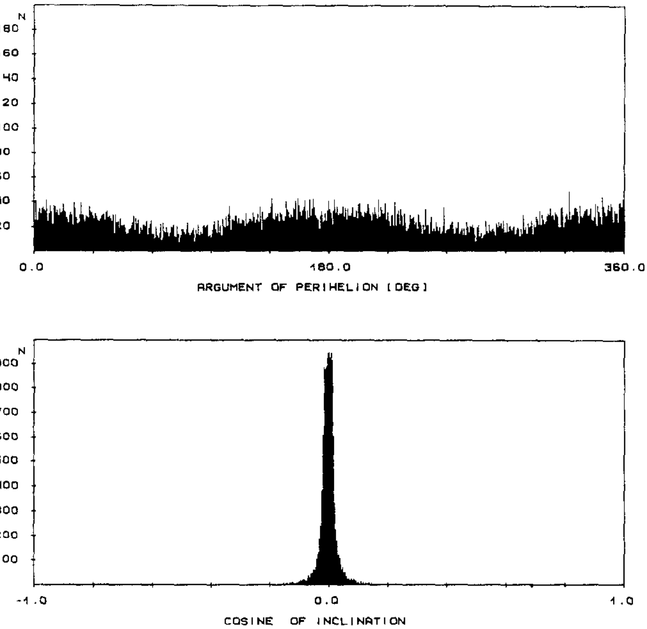

Fig. 1. Orbital elements distributions for potentially observable part of the Oort cloud.

is perfectly reproduced. Analyzing examples of long term orbital evolution we came to conclusion that the orbits of observable (or almost observable) comets, with eccentricity greater than 0.99, have short period fluctuations of negligible amplitude.

We thus decided to use the first order averaged theory in our calculations. We used both an analytical solution and the numerical integration of Hamiltonian averaged equations. The results of these two approaches were in good agreement except for certain sets of orbital elements for which the series in the analytical solution converged extremely slowly. In order to follow the orbital evolution step by step for long time intervals, dealing with arbitrary values of elements, numerical integration of the averaged equations appears to be the simplest and the most effective method.

In their papers, Matese and Whitman used a Monte-Carlo simulation of the population of the Oort cloud comets, with orbital elements distributed according to uniform probability densities to produce the observable comets. After a random selection of all initial parameters for a given comet, they determinated whether the galactic tidal perturbation was strong enough to decrease the comet's perihelion distance from initially more than 15 AU to less than 5 AU after one orbital period. If that was the case, the comet was marked as 'observable'. In the second paper Matese and Whitman (1992)

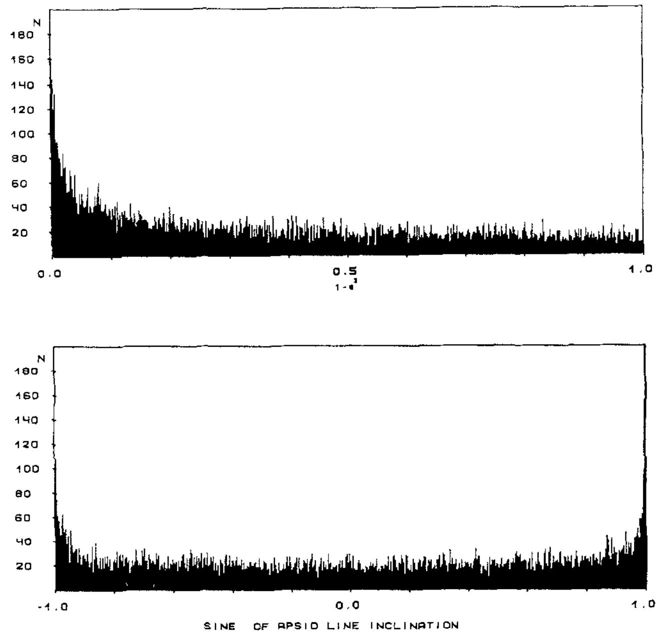

Fig. 2. Orbital elements distributions for potentially observable part of the Oort cloud.

presented distributions of argument of perihelion, the inclination, and the inclination of line of apsides for the observable comet population, recorded at the end of the one orbital period interval. It means, that this population consists of the majority of comets with q decreasing but there is a significant number of comets with q increasing.

2. Results

Having had the opportunity to observe the individual orbital evolution of arbitrary comets in the cloud, we concluded that treating the orbital elements of comets in the Oort cloud as independent, random variables, with uniform probability densities, may not be the best approximation. Looking at the plots presented in Paper A, it is easy to observe the coincidence of rapid changes in certain pairs of orbital elements during their long term evolution under the influence of the galactic disk tide. For example, in a typical situation the inclination reaches its lowest value when the argument of perihelion equals 90° or 270° exactly. At the same moment the eccentricity reaches its maximum value. Due to the periodic nature of this evolution, a similar situation repeats many times. Moreover, looking at the evolutionary paths of the argument of perihelion one can easily guess that its value recorded at random moments, will more probably be in the vicinity of 180° than the vicinity of 90°.

Taking all this into account, we think that the long term perturbation of the galactic tide induces a deformation in the distributions of orbital elements and their mutual dependence. This point of view may be looked upon as an opposite extreme to that proposed by Matese and Whitman. They allow the galactic tide to operate only during one orbital period of a comet (typically several million years). This treatment assumes that over longer time intervals the cometary cloud is completely and isotropically randomized by stellar and GMC impulses. There is no evidence that those impulses are effective enough to erase any trace of the long term influence of galactic tides.

We present here some statistical characteristics of the observable comet population in the absence of randomizing impulses for time intervals comparable to the period of the long term variation of orbital elements. For the real cometary cloud the orbital element distributions are the result of a balance between tidal effects and those randomizing impulses. Moreover, it should be noted that each individual stellar or GMC encounter is extremely nonisotropic in its effects, as one can see for example in a paper by P.A. Dybczyński presented as a poster at this conference.

As a first step, we observed changes in the distributions of orbital elements in the cloud over a long time interval. We started with the uniform distributions proposed by Matese and Whitman ($a = 30000$ AU, $1 - e^2 \in (0, 1)$) and followed the orbital evolution of each comet for 1 Gyr.

The resulting distributions remain almost uniform, showing only small deviations. Then we determinated the distributions of elements of the observable part of the cloud. During the integration spanning 1 Gyr we marked as potentially observable all those comets which – at some time – had a perihelion distance less than the 5 AU threshold. In this manner we divided the Oort cloud into potentially observable and non-observable parts. Figs. 1(a,b), 2(a,b) present the distributions of ω, $\cos i$, $\sin b$, and $1 - e^2$ for the observable part of the cometary population, recorded at an arbitrary moment of time. The vast majority of orbits have inclinations close to 90°, contrary to the observed cometary orbits. This fact can be easily explained. The inclination of the majority of observable comets follows very similar evolutionary paths. An example of such a variation is shown in Paper A (Fig. 4). The inclination remains almost constant and equal to 90° most of the time, with the exception of the very short time interval when the perihelion distance reaches its minimum (and when a comet may be observed).

Finally, we obtained a set of element distributions recorded at the moment that the comets first crossed the 5 AU observability barrier. As shown in Fig.3(a,b,c), among the observed comets almost all inclinations are present, except for the smallest values. However, the decrease in the vicinity of 90°

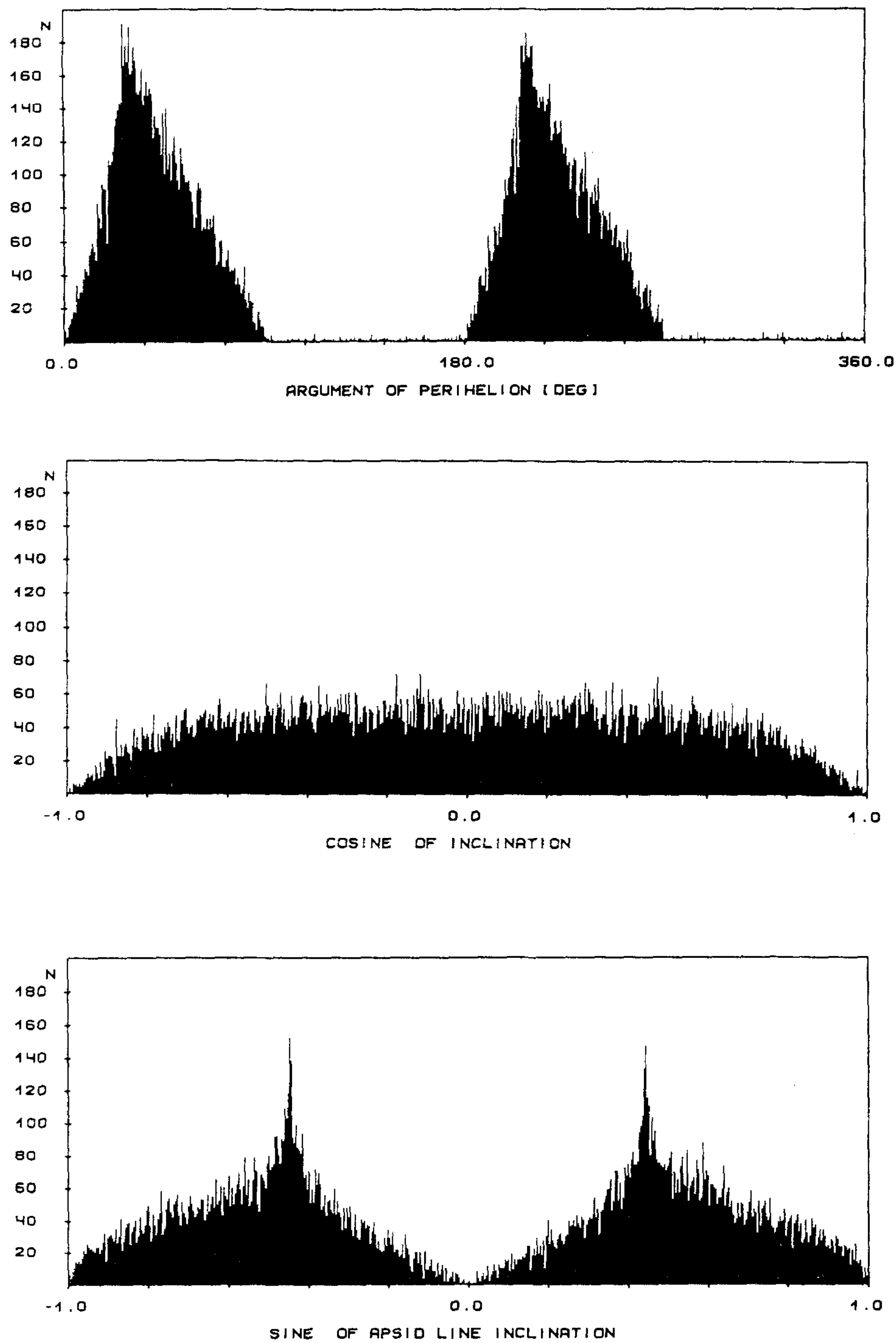

Fig. 3. Orbital elements distributions for the Oort cloud comets at the moment of crossing 5 AU threshold in the perihelion distance.

found by Matese and Whitman is absent here. The ω distribution shows a similar shape to that obtained by Matese and Whitman, but here almost half of the interval is vacant. This results from the fact that in contrary to Matese and Whitman we recorded osculating elements at the moment of q decreasing below 5 AU limit. Detailed comparison of the $\sin b$ distributions reveals remarkable differences in their shapes.

In our case the sine of the inclination of line of apsides concentrates around the critical values described in Paper A (approximately $\pm 27°$). In the Matese and Whitman case the extreme values are $\pm 45°$ because of the strong selection effect as they stated in their paper (1992).

3. Conclusions

In our opinion, all deformations and mutual dependences present in these distributions of orbital elements should also be present in the initial distributions used by Matese and Whitman. We plan to repeat their calculations with this modification. Such a work is in progress and we hope to obtain first results soon.

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